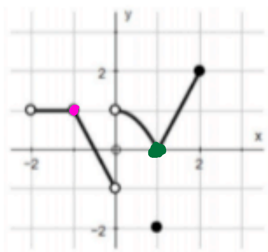
Period 1, 8/28/24
Questions to review
1, 7, 10, 11, 13, 15, 16, 17, 18, 20, 26, 28, 34, 47

2. Use the function $g(x)$ defined and graphed below to answer the following questions.



$$g(x) = \begin{cases} 1, & -2 < x < -1 \\ -2x - 1, & -1 < x < 0 \\ 1 - x^2, & 0 < x < 1 \\ -2, & x = 1 \\ 2x - 2, & 1 < x \le 2 \end{cases}$$

a) Does $g(1)$ exist?

b) Does $\lim_{x \to 1} g(x)$ exist?

c) Does $\lim_{x \to 1} g(x) = g(1)$?

d) Is g continuous at $x = 1$?

e) Is g defined at $x = -1$?

f) Is g continuous at $x = -1$?

[ ] g) For what values of x is g continuous?

$(-2,-1) \cup (-1,0) \cup (0,1) \cup (1,2]$

h) What value should be assigned to $g(-1)$ to make the extended function continuous at $x = -1$?

$g(-1) = 1$

i) What new value should be assigned to $g(1)$ to make the new function continuous at $x = 1$?

$g(1) = 0$

j) Is it possible to extend g to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

iT is NoT a Removable discontinuity

6. Select values of the continuous function $f(x)$ are shown below.

x	-2	0	2	4	6	8
$f(x)$	-20	-1	-1	-25	1	5

a) For each of the following, conclude if $f(x)$ reaches the selected values and if so, list the interval(s) where it could occur. If not sure, explain why.

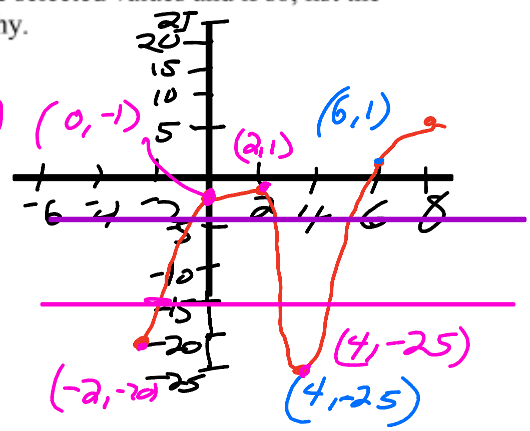
IVT

i. $f(x) = 0$ [4,6]

ii. $f(x) = -15$ (-2,0) ∪ (2,4) ∪ (4,6)

iii. $f(x) = 7$

F(x) is continious No



b) At least how many times does $f(x) = -4$?

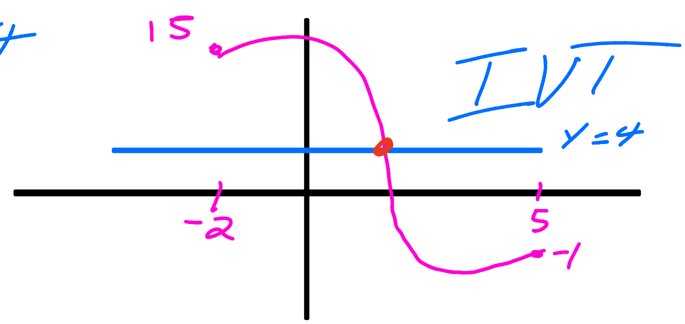
at least 3

(-2,0) ∪ (2,4) ∪ (4,6)

so continious

8. The function $g(x)$ is differentiable function on the interval $[-2,5]$. If $g(-2) = 15$ and $g(5) = -1$, can you conclude that $g(x)$ equals 4? If so, on what interval and how do you know?

cross 4



IVT

5. The function f is defined by $f(x) = \begin{cases} x^2 - 2x + 3 \text{ Para} & \text{if } x \le 1 \\ -2x + 5 \text{ Line} & \text{if } x > 1 \end{cases}$

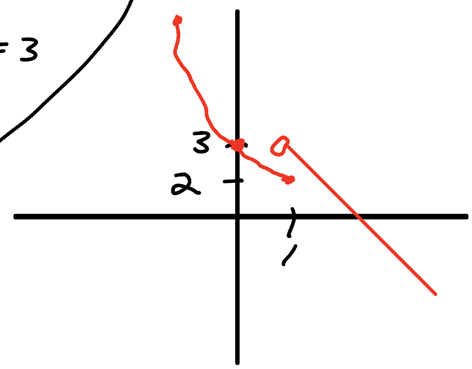
$F(1) = 2$

(a) Is f continuous at $x = 1$?
(b) Use the definition of continuity to explain your answer.

$\lim_{x \to 1^+} F(x) = -2x + 5 = -2(1) + 5 = 3$

$\lim_{x \to 1^-} F(x) = (1)^2 - 2(1) + 3 = 2$

$\lim_{x \to 1} F(x) = \text{DNE}$



Note: The statement $\lim_{x \to c} f = \infty$ **does not** mean that the **limit exists**. On the contrary, it tells you **how the limit fails to exist** by denoting the unbounded behavior of $f(x)$ as x approaches c.

1a Find the vertical asymptotes (if any) of the graph of the function. find the limit of the function as x approaches from the left and the right of each asymptote, then find the value of the limit at each asymptote.

$$f(x) = \frac{x^2 - 1}{2x - 4} = \frac{\overset{+}{(x-1)}\overset{+}{(x+1)}}{2(x-2)}$$

$F(1) = 0$

$F(-1) = 0$

$\lim_{x \to 2^+} F(x) \Rightarrow +\infty$ $\quad x > 2 \quad \frac{+ \cdot +}{+} = +$ $\quad x = 2$

$\lim_{x \to 2^-} F(x) \Rightarrow -\infty$ $\quad x < 2 \quad \frac{+ \cdot +}{-} = -$

1b

Find the vertical asymptotes (if any) of the graph of the function. find the limit of the function as x approaches from the left and the right of each asymptote, then find the value of the limit at each asymptote.

$$h(x) = \frac{1-x}{x^2-4x+3} = \frac{-1(x-1)}{(x-1)(x-3)}$$

at $x=1$ hole

$x=3$ asymptote

$$\lim_{x \to 1} h(x) = \frac{-1}{x-3} = \frac{-1}{-2} = \frac{1}{2}$$

$$\lim_{x \to 3^+} h(x) = \frac{-}{+ \cdot +} = \frac{-}{+} = -\infty$$

$$\lim_{x \to 3^-} h(x) = \frac{-}{+ \cdot -} = \frac{-}{-} = +\infty$$

find the value of the limit at

$$g(x) = \frac{1}{(x+1)^2}$$

$$\lim_{x \to -1} \frac{1}{(x+1)^2} = \frac{1}{+RSN} = \infty = DNE$$

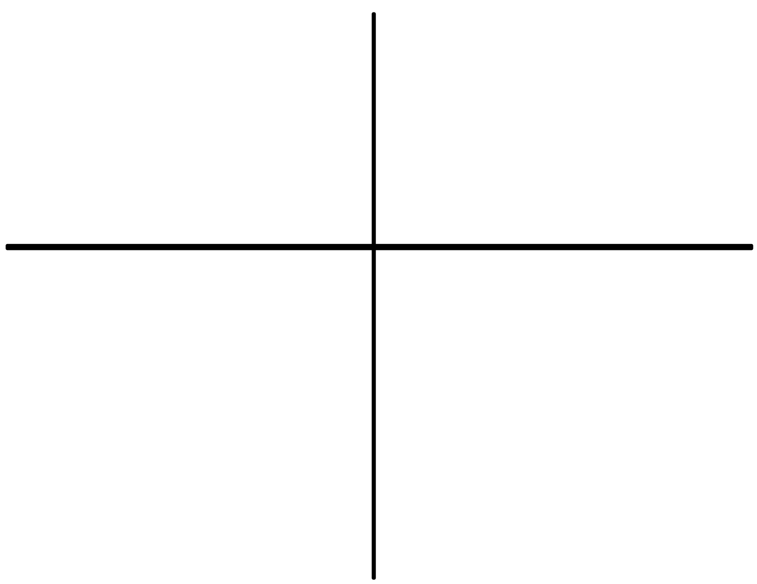
Theorem Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x\to c} f(x) = \infty \quad \text{and} \quad \lim_{x\to c} g(x) = L.$$

1. **Sum or Difference**: $$\lim_{x\to c}[f(x) \pm g(x)] = \infty$$
2. **Product**: $$\lim_{x\to c}[f(x)g(x)] = \infty,\ L > 0$$ $$\lim_{x\to c}[f(x)g(x)] = -\infty,\ L < 0$$
3. **Quotient**: $$\lim_{x\to c}\frac{g(x)}{f(x)} = 0$$

Similar properties hold for one-sided limits and for **functions** for which the limit of $f(x)$ as x approaches c is $-\infty$.

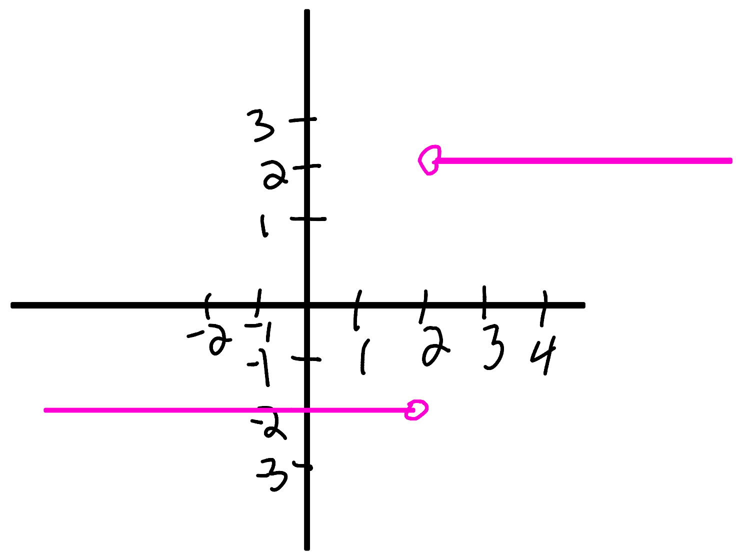


$$\lim_{x\to 2}\frac{2x-4}{|x-2|} = \lim_{x\to 2}\frac{2(x-2)}{|x-2|} = DNE$$

$$\lim_{x\to 2^+}\frac{2x-4}{|x-2|} = \underline{\quad 2 \quad}$$

$$\lim_{x\to 2^-}\frac{2x-4}{|x-2|} = \underline{\quad -2 \quad}$$

x	y
-1	$\frac{-6}{3} = -2$
0	$\frac{-4}{2} = -2$
1	$\frac{-2}{1} = -2$
3	$\frac{2}{1} = 2$
4	$\frac{4}{2} = 2$
5	$\frac{6}{3} = 2$



64. Given a polynomial $p(x)$, is it true that the graph of the function given by $f(x) = \dfrac{p(x)}{x-1}$ has a vertical asymptote at $x = 1$? Why or why not?

$P(x) = x^2 + 3x + 9$

$$\lim_{x\to 1}\frac{x^2+3x+9}{x-1} = \frac{13}{\pm RSN} = \pm\infty \quad \text{Vertical asy}$$

$P(x) = x^2 + 3x - 4$ Hole at 1

$$\lim_{x\to 1}\frac{x^2+3x-4}{x-1} = \lim_{x\to 1}\frac{(x+4)\cancel{(x-1)}}{\cancel{(x-1)}} = 1+4 = 5$$

$4x^2 + 17x + 15$

$4 \cdot 15 = 60$

$6 + 10 = 16$

$-6 + -10 = -16$

$5 + 12 = 17 \Rightarrow 4x^2 + 5x + 12x + 15$

$x(4x+5) + 3(4x+5)$

$(4x+5)(x+3)$

B #16, B #1, 23, 26, C #1, A #29, D #20

#32 #37 #13 #50

#34 #48 #26

#35

$\frac{5}{x-7} + 1 = \frac{8}{x-7} \qquad x \neq 7$

$\frac{5}{x-7} + \frac{x-7}{x-7} = \frac{8}{x-7}$

$\frac{5+x-7}{x-7} = \frac{8}{x-7}$

$x = 10 \qquad \cancel{(x-7)}\frac{x-2}{\cancel{x-7}} = \frac{8}{\cancel{x-7}}\cancel{(x-7)}$

$\left(\frac{xy^6}{x^3y}\right)^{-2}$

$\left(\frac{y^5}{x^2}\right)^{-2}$

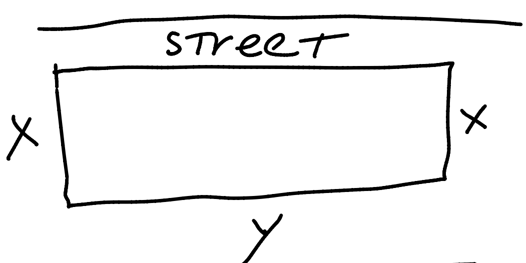
$\frac{y^{-10}}{x^{-4}} = \frac{x^4}{y^{10}}$

$F(x) = \frac{7}{x-8}$ $\quad g(x) = \frac{7}{3x}$

$$(F \circ g)(x) = F(g(x)) = \frac{7}{g(x)-8} = \frac{7}{\frac{7}{3x} - \frac{8 \cdot 3x}{1 \cdot 3x}}$$

$$\frac{7}{1} \cdot \frac{3x}{7-24x} \Leftarrow \frac{\frac{7}{1}}{\frac{7-24x}{3x}}$$

$$\frac{21x}{7-24x}$$



$\frac{-b}{2a}$ = vertex

$\frac{-324}{2(2)} = \frac{-324}{-4} = 81$

$x = 81$

$y = 324 - 2(81)$

$= 162$

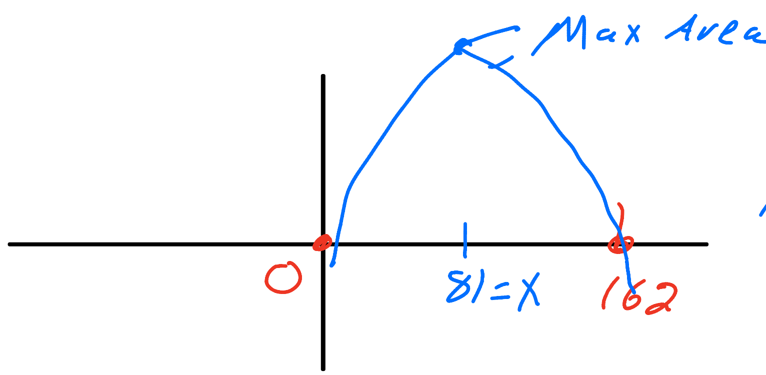
$x + y + x = 324 \Rightarrow y = \boxed{324 - 2x}$

Area $= x \cdot y$

Area $= -2x^2 + 324x + 0$

Area $= x(324 - 2x) = 2x(162 - x)$

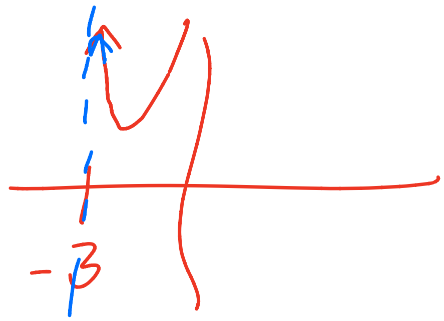
Parabola opens down



Area $= 162 \cdot 81$

13,122

$x \to -3^+$ $\quad F(x) = +\infty$



$$g(x) = \frac{4x^2}{2x^2+1} \qquad \lim_{x \to \infty} \frac{4x^2}{2x^2+1} = \frac{4\cancel{x^2}}{2\cancel{x^2}} = 2$$

$$\log_{10}\sqrt{10} = \log_{10} 10^{\frac{1}{2}} = \frac{1}{2}\log_{10} 10 = \frac{1}{2}\cdot 1 = \frac{1}{2}$$

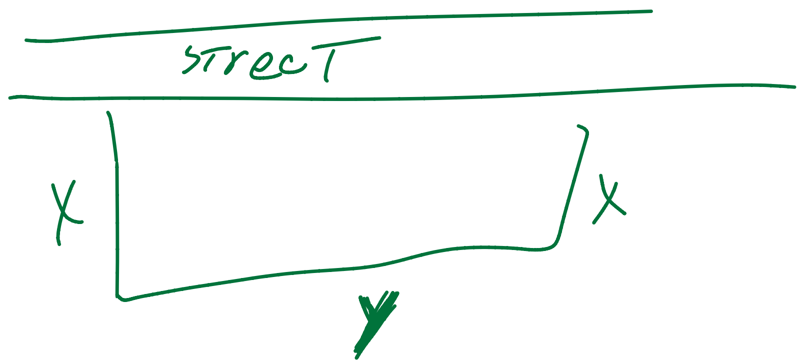
$$\log_b \frac{xy^3}{z^8} = \log_b xy^3 - \log_b z^8$$

$$\log_b x + \log_b y^3 - \log_b z^8$$

$$\log_b x + 3\log_b y - 8\log_b z$$

$$\sin^2 x\,(1+\cot^2 x) = \sin^2 x\left(1 + \frac{\cos^2 x}{\sin^2 x}\right)$$

$$\sin^2 x\left(\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}\right) = \sin^2 x\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) = \cancel{\sin^2 x}\cdot\frac{1}{\cancel{\sin^2 x}}$$



$$x+y+x=252 \Rightarrow y=252-2x$$

$$\text{Area}=xy=x(252-2x)=252x-2x^2$$

$$\text{Area}=-2x^2+252x+0$$

$$a=-2 \quad b=252$$

Vertex x-value $= \dfrac{-b}{2a}=\dfrac{-252}{2(-2)}$

$(63, 7938)$

$$\text{Area}=-2(63)^2+252(63)$$

7938

$-1-6+1+6 \Rightarrow x=-1$

$$x^3-6x^2-x+6=0$$

$$1-6-1+6=0$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & -1 & 6 \\ & & 1 & -5 & -6 \\ \hline & 1 & -5 & -6 & \boxed{0} \end{array}$$

$1x^2-5x-6$

$$x^3-6x^2-x+6=(x-1)(\quad ? \quad)$$

$$=(x-1)(x^2-5x-6)$$

$$(x-1)(x-6)(x+1)$$

$$x=1, 6, -1$$

$$\underbrace{6^3-6(6)^2}_{0}+\underbrace{-6+6}_{0}=0$$

$\cos 2x$ $\quad\quad\quad\quad$ $\cos(a+b) = \cos a \cos b - \sin a \sin b$

$$\cos(x+x) = \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\underbrace{\sin^2 x + \cos^2 x}\ = 1$$

$$\cos^2 x - \sin^2 x$$
$$\cos^2 x - \sin^2 x - 1 + 1$$
$$\cos^2 x - \cancel{\sin^2} x - 1 + \cancel{\sin^2} x + \cos^2 x = 2\cos^2 x - 1$$
$$\cos^2 x - \sin^2 x - (\sin^2 x + \cos^2 x) + 1$$
$$\cancel{\cos^2} x - \sin^2 x - \sin^2 x \cancel{- \cos^2} x + 1$$
$$1 - 2\sin^2 x$$

$$\left(\frac{xy^3}{x^4 y}\right)^{-2} = \left(\frac{y^2}{x^3}\right)^{-2} = \frac{y^{-4}}{x^{-6}} = \frac{x^6}{y^4}$$

$$\frac{5\,x(x-1)}{(x+1)(x-1)} + \frac{6(x+1)}{(x-1)(x+1)} - \frac{10}{x^2-1}$$

$$(x+1)(x-1) = x^2-1$$

$$\frac{5x^2-5x}{x^2-1} + \frac{6x+6}{x^2-1} - \frac{10}{x^2-1}$$

$$\frac{5x^2-5x+6x+6-10}{x^2-1} = \frac{5x^2+x-4}{x^2-1}$$

$5x^2+x-4$

$5 \cdot -4 = -20$

$5 + -4 = 1$

$\underbrace{5x^2+5x}_{5x(x+1)} \underbrace{-4x-4}_{-4(x+1)}$

$(x+1)(5x-4)$

$$\frac{(5x-4)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)} \qquad x \neq 1, -1$$

$$\frac{5x-4}{x-1}$$

$$\frac{\csc x \sec x}{\tan x} = \frac{\frac{1}{\sin x} \cdot \frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{\frac{1}{\sin x \cos x}}{\frac{\sin x}{\cos x}}$$

$$\frac{1}{\sin x \cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x} = \frac{1}{\sin^2 x} = \csc^2 x$$

$$x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$x \neq \frac{k\pi}{2}$$

$$F(x) = 3 - 19x \qquad \lim_{h \to 0} \frac{F(a+h) - F(a)}{h}$$

$$F(a) = 3 - 19a$$

$$F(a+h) = 3 - 19(a+h)$$

$$\lim_{h \to 0} \frac{[3 - 19(a+h)] - (3 - 19a)}{h} = \frac{\cancel{3} - \cancel{19a} - 19h - \cancel{3} + \cancel{19a}}{h}$$

$$\frac{-19\cancel{h}}{\cancel{h}} = -19$$

$$F(x)=6x^2$$

$$F(x+h)=6(x+h)^2=6(x^2+2xh+h^2)$$

$$\frac{F(x+h)-F(x)}{h}=\frac{6(x^2+2xh+h^2)-6x^2}{h}$$

$$\frac{\cancel{6x^2}+12xh+6h^2-\cancel{6x^2}}{h}$$

$$\frac{\cancel{h}(12x+6h)}{\cancel{h}}=12x+6h$$

$$6(2x+h)$$

$$F(x)=9x^2$$

$$F(x+h)=9(x+h)^2=9(x^2+2xh+h^2)$$

$$\frac{9(x^2+2xh+h^2)-9x^2}{h}=\frac{\cancel{9x^2}+18xh+9h^2-\cancel{9x^2}}{h}$$

$$\frac{\cancel{h}(18x+9h)}{\cancel{h}}=9(2x+h)$$